

# DIQUARK CONDENSATION IN DENSE MATTER A LATTICE PERSPECTIVE

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We review efforts to study, using the methods of lattice field theory, the phenomenon of diquark condensation via BCS pairing at a Fermi surface, which has been proposed as a mechanism for color superconductivity in dense quark matter. The particular models studied are the Gross-Neveu model and SU(2) lattice gauge theory; in both cases evidence for superfluidity at high density is presented. The behaviour expected for quarks in both fundamental and adjoint representations of SU(2) is contrasted.

## 1 Introduction

At low temperatures, fermionic matter becomes degenerate; ie. all single particle states are occupied up to some sharply defined scale, the *Fermi energy*, which characterises allowed physical processes. In a normal Fermi liquid, excitations above the Fermi energy can have arbitrarily small energy. However, if there is an attractive interaction between fermion pairs, and the Fermi energy is large enough, then the normal state should be unstable with respect to the opening of a gap at the Fermi surface. In field-theoretic language, this gap is signalled by a non-vanishing condensate formed from two elementary fermion fields. If our fermions are quarks, then we have a *diquark condensate*  $\langle qq \rangle \neq 0$ . As reviewed below, diquark condensation can lead to new and more interesting ground states. In this talk we will summarise our efforts in, and future prospects for, probing this phenomenon using the methods of lattice field theory. Throughout we will be attempting to explore the zero temperature limit, although this may be difficult to reach in practice on finite volumes.

When considering the possible diquark condensates that may form, various questions present themselves. Firstly, if the quarks carry a gauge charge, is the condensate gauge invariant? If so, then the state with  $\langle qq \rangle \neq 0$  breaks some global symmetry of the Lagrangian, implying eg. that excitations carry indefinite baryon number. This is the phenomenon of *superfluidity*, which occurs in condensed matter physics at milliKelvin temperatures for He<sup>3</sup>. The systems we have studied to date are expected to exhibit superfluidity. However, if the condensate breaks a local symmetry, then the system will be a *superconductor*, and the diquark condensate a direct analogue of the Cooper

pairing found in metals at low temperature. Since the quarks of QCD transform under a non-Abelian symmetry, the phenomenon is still more exotic: *color superconductivity* has been postulated as a dynamical embodiment of the Higgs mechanism, making some or all of the gluons massive at high baryon density.<sup>1,2,3</sup> Secondly, is the condensate a spacetime scalar? Naively one might expect so, but examples of rotationally non-invariant condensates are known in condensed matter physics ( $\text{He}^3\text{A}$ ), and have been postulated in QCD.<sup>2</sup> A parity violating diquark condensate is another possibility.

The most crucial consideration is that the condensate must respect the Pauli Exclusion Principle, so that its wavefunction is antisymmetric under exchange of all available quantum numbers, which in the case of quarks are spacetime, color and flavor. This has the consequence that the ground state may be extremely sensitive to the number of light flavors present,<sup>2</sup> or as we shall see below, to the representation of the color group carried by the quarks. This impact of the Exclusion Principle makes the phenomenon of intrinsic theoretical interest, and may even underlie the known difficulties in simulating QCD with non-zero baryon density.<sup>4</sup>

## 2 Four Fermi Models

The first model we will consider is referred to as either the “Gross-Neveu” (GN) model in 2+1, or the “Nambu – Jona-Lasinio” (NJL) model in 3+1 dimensions, and is a relativistic generalisation of the BCS model originally used to describe superconductivity. It has the following Lagrangian density in continuum notation:

$$\mathcal{L} = \bar{\psi}(\not{\partial} + m)\psi - g^2 [(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\vec{\tau}\psi)^2], \quad (1)$$

where  $\psi$  is an isodoublet transforming under a global axial  $\text{SU}(2)_L \otimes \text{SU}(2)_R$  symmetry, which is exact in the chiral limit  $m \rightarrow 0$ :

$$\psi_L \mapsto U\psi_L, \bar{\psi}_L \mapsto \bar{\psi}_L U^\dagger; \quad \psi_R \mapsto V\psi_R, \bar{\psi}_R \mapsto \bar{\psi}_R V^\dagger. \quad (2)$$

$\mathcal{L}$  is also invariant under a global  $\text{U}(1)_V$  of baryon number:

$$\psi \mapsto e^{i\alpha}\psi; \quad \bar{\psi} \mapsto \bar{\psi}e^{-i\alpha}. \quad (3)$$

Let us review some properties of the model. First, for sufficiently strong coupling  $g^2$ , the axial symmetry spontaneously breaks to a  $\text{SU}(2)_V$  of isospin by the formation of a chiral condensate  $\langle\bar{\psi}\psi\rangle$ . The spectrum in the chirally broken phase contains massive “baryons” – the elementary fermions, and  $\psi\bar{\psi}$  composite “mesons”, which include three light Goldstone pions. The chiral condensate sets the dynamical mass scale  $M/\Lambda$  for the model, which can thus

be taken to zero at some critical  $g_c^2$ , defining a continuum limit. A special feature of the GN model is that the continuum limit defines an interacting field theory, whereas that of the NJL model is logarithmically trivial.<sup>5</sup> For the present purposes, the most interesting feature of the models is that they can be formulated on a lattice, and simulated with baryon chemical potential  $\mu \neq 0$  by standard Monte Carlo methods;<sup>6</sup> the reasons for this are subtle and only recently understood.<sup>7</sup> It is found that for  $\mu$  greater than some  $\mu_c$  of order the baryon mass the symmetry (2) is restored.

In our work<sup>8</sup> we have focussed on the formation at large  $\mu$  (for non-interacting fermions  $\mu$  is precisely the Fermi energy) of a diquark condensate  $\langle qq \rangle$ , where in continuum notation the diquark wavefunction in 2+1 dimensions reads

$$qq = \psi^{tr}(\mathcal{C}\gamma_5) \otimes \tau_2 \otimes \tau_2 \psi. \quad (4)$$

Here  $\mathcal{C}$  is the charge conjugation matrix, and the  $\mathcal{C}\gamma_5$  structure ensures that the condensate is a scalar. The first of the  $\tau_2$  matrices operates on an implicit flavor space due to the species doubling generic to lattice fermions, and the second on the explicit isospin indices. Since all three operators are anti-symmetric matrices, the overall antisymmetry of the wavefunction is ensured. This condensate respects the axial symmetry (2) but spontaneously breaks  $U(1)_V$  (3).

### 2.1 Two-Point Function Approach

Our first attempt to look for a signal for  $\langle qq \rangle \neq 0$  was via the asymptotic behaviour of the diquark propagator, which by the cluster property should be proportional to the square of the condensate:

$$G(x) = \langle qq(0)\bar{q}\bar{q}(x) \rangle = \langle qq(0)\bar{q}\bar{q}(x) \rangle_c + \langle qq \rangle \langle \bar{q}\bar{q} \rangle \Rightarrow \lim_{x \rightarrow \infty} G(x) = |\langle qq \rangle|^2. \quad (5)$$

The condensate should thus manifest itself as a plateau in the large- $t$  behaviour of the timeslice propagator. Results<sup>8</sup> for  $G(t) = \sum_{\vec{x}} G(\vec{x}, t)$  from a  $16^2 \times 40$  lattice, with  $\mu = 0.8 > \mu_c$ , are shown in Fig. 1, clearly showing a stable plateau when compared to the same quantity obtained for non-interacting fermions, shown with closed symbols. Taking the square root of the plateau height as a measure for  $\langle qq \rangle$  results in the plot shown in Fig. 2, where chiral symmetry restoration at  $\mu_c \simeq 0.65$  followed by a rapid rise in baryon number density is also clearly visible. We see a dramatic increase in  $\langle qq \rangle$  proceeding from the low density chirally broken phase to the high density chirally symmetric phase, suggesting that diquark condensation is taking place across the transition, as revealed by long range order in the timelike direction. This

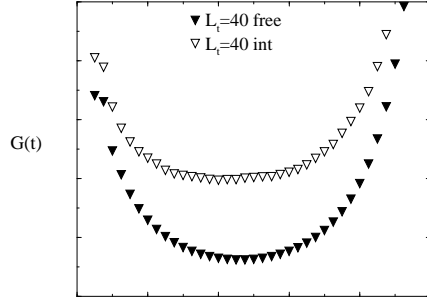


Figure 1.

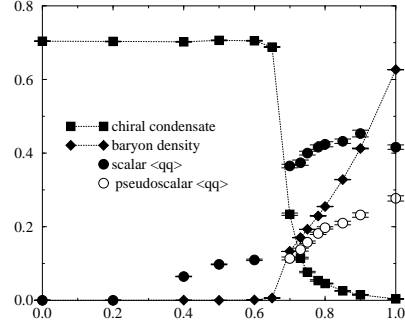


Figure 2.

result is consistent with the notion of competition between chiral and diquark condensates.<sup>2</sup> We have also observed signals both for a small scalar diquark condensate in the broken phase, and for a non-vanishing *pseudoscalar* condensate in the dense phase, implying spontaneous breaking of parity; both these may well be finite volume artifacts, however.

Unfortunately, data from lattices with spatial volume  $L_s^2$  varying from  $8^2$  to  $24^2$  do not support a naive application of Eq. (5), which suggests the plateau height should be an extensive quantity. Indeed, the plateau height saturates for  $L_s \geq 16$ , a result which may be due to the influence of Goldstone fluctuations, which would wash out the signal in the absence of an external source, or from the relatively small number of participating  $qq$  states in the vicinity of the Fermi surface on these moderate systems.

## 2.2 One-Point Function Approach

In an attempt to clarify matters we have also performed direct measurements of the diquark one-point function  $\langle qq \rangle$ . By rewriting the fermionic action  $S_{ferm} = \bar{\psi} M \psi$  in the *Gor'kov representation*, it is possible to add explicit diquark source terms:

$$S_{ferm} = (\bar{\psi}, \psi^{tr}) \begin{pmatrix} \bar{j}\tau_2 & \frac{1}{2}M \\ -\frac{1}{2}M^{tr} & j\tau_2 \end{pmatrix} \begin{pmatrix} \bar{\psi}^{tr} \\ \psi \end{pmatrix} \equiv \Psi^{tr} \mathcal{A}[j, \bar{j}] \Psi; \quad (6)$$

$$Z[j, \bar{j}] = \langle \text{Pf} \mathcal{A}[j, \bar{j}] \rangle.$$

The diquark condensate is then defined by

$$\langle qq \rangle = \frac{1}{V} \frac{\partial \ln Z}{\partial j} \Big|_{j, \bar{j}=0} = \lim_{j, \bar{j} \rightarrow 0} \frac{1}{V} \left\langle \frac{1}{2} \text{tr} \left\{ \mathcal{A}^{-1} \begin{pmatrix} & \\ & \tau_2 \end{pmatrix} \right\} \right\rangle, \quad (7)$$

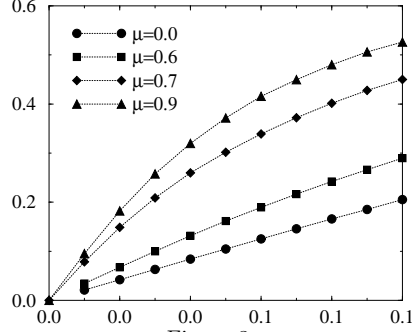


Figure 3.

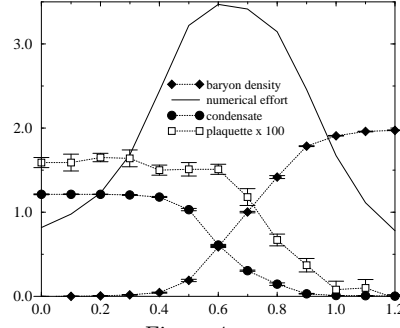


Figure 4.

which is straightforward to implement. Our results for  $\langle qq(j) \rangle$  in the GN model from a  $16^2 \times 24$  lattice are shown in Fig. 3. We see a large jump in the signal as the chiral transition takes place; at low density the signal is approximately linear in  $j$ , whereas at high density there is a marked curvature. In neither case, however, is there convincing evidence for a non-zero intercept as  $j \rightarrow 0$ .

It should be stressed that these measurements are “quenched” in the sense that the diquark source term is not included in the Monte Carlo update algorithm. We are currently attempting full simulations with  $j \neq 0$  in the expectation that finite volume effects will be easier to interpret.<sup>9</sup> Inclusion of the source term will also enable estimates of disconnected contributions to the two-point function, which may have a better overlap with the Goldstone mode if one exists. It will also be valuable to develop spectroscopy techniques in the Gor’kov representation, since in the presence of a gap the eigenstates of the transfer matrix will be a superposition of  $q$  and  $\bar{q}$  states. It is safe to say that much more work is needed before the apparently straightforward exercise presented by four fermi models is complete.

### 3 Two Colors QCD

#### 3.1 Fundamental Quarks

SU(2) gauge theory with staggered lattice fermions in the fundamental representation does not suffer from the difficulties associated with simulating dense QCD, since it can be shown that  $\det M$  is real<sup>a</sup> and positive for all  $\mu$ <sup>10</sup>. In Fig. 4 we show results from runs on a  $4^4$  lattice, with  $N_f = 4$  physical flavors

<sup>a</sup>This follows readily from  $\det M = \det \tau_2 M \tau_2 = \det M^*$ .

of fermion and  $m = 0.2$ , in the strong gauge coupling limit as a function of  $\mu$ . For  $\mu < 0.4$ , the system remains essentially unchanged, but for  $0.4 < \mu < 0.9$  the chiral condensate  $\langle \bar{\psi}\psi \rangle$  smoothly decreases to zero, at the same time as the baryon density  $n \equiv \langle \bar{\psi}\gamma_0\psi \rangle$  increases from zero to its saturation value of two quarks per lattice site. The numerical effort required, as measured by the number of computer iterations required to invert  $M$ , rises steeply in the crossover region. Most interestingly, the average plaquette decreases as density rises, until at saturation it assumes the expected quenched value of zero. This is a signal of *Pauli blocking*; at high density virtual  $q\bar{q}$  pairs are suppressed for kinematical reasons, and color screening via vacuum polarisation thus reduced.

A physically appealing way of understanding the role of the complex phase of  $\det M$  in gauge theories is in terms of *conjugate quarks*.<sup>11</sup> Monte Carlo simulations of QCD demand a real functional measure, and therefore update using  $\det MM^\dagger = \det M \det M^*$ . The  $M$  describes quarks  $q$  in the  $\mathbf{3}$  representation of  $SU(3)$ , the  $M^*$  conjugate quarks  $q^c$  in the  $\bar{\mathbf{3}}$ . Usually conjugate quarks are regarded as anti-quarks of a different flavor, but once  $\mu \neq 0$  it can be checked that both  $q$  and  $q^c$  couple to  $\mu$  in the same way and hence carry positive baryon number. Therefore simulations with a real measure can describe gauge singlet  $qq^c$  bound states; in particular a pseudo-Goldstone state, the *baryonic pion* can form,<sup>12</sup> and is the lightest baryon in the spectrum, since by the usual PCAC arguments its mass is expected to vanish as  $\sqrt{m}$  in the chiral limit. Now, simple energetic arguments suggest that observables should start to show  $\mu$ -dependence for  $\mu \geq \mu_o \simeq m_{lb}$ , where  $m_{lb}$  is the lightest baryon mass. The presence of an unphysical light baryon therefore plays havoc with the physics of  $\mu \neq 0$ .<sup>b</sup>

For  $SU(2)$  gauge theory, however, baryonic pions are not unphysical. Consider the kinetic term for staggered lattice fermions:

$$S_{kin} = \frac{1}{2} \sum_{x, \mu=0,3} \eta_\mu(x) [\bar{\chi}(x) U_\mu(x) \chi(x + \hat{\mu}) - \bar{\chi}(x) U_\mu^\dagger(x - \hat{\mu}) \chi(x - \hat{\mu})]. \quad (8)$$

We can identify both  $U(1)_V$  and  $U(1)_A$  global symmetries, which may be associated with baryon number and axial charge conservation respectively:

$$U(1)_V : \chi \mapsto e^{i\alpha} \chi \quad \bar{\chi} \mapsto \bar{\chi} e^{-i\alpha} \quad U(1)_A : \chi \mapsto e^{i\beta\varepsilon} \chi \quad \bar{\chi} \mapsto \bar{\chi} e^{i\beta\varepsilon}, \quad (9)$$

where  $\varepsilon(x) = (-1)^{x_0+x_1+x_2+x_3}$ . However, by defining new fields

$$\bar{X}_e = (\bar{\chi}_e, -\chi_e^{tr} \tau_2), \quad X_o^{tr} = (\chi_o^{tr}, \bar{\chi}_o \tau_2), \quad (10)$$

<sup>b</sup>For four fermi models,  $q\bar{q}$  states are much lighter than  $qq^c$  due to disconnected diagrams.<sup>7</sup>

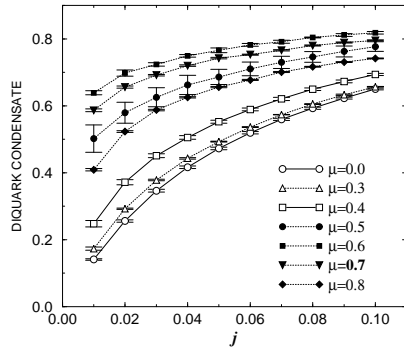


Figure 5.

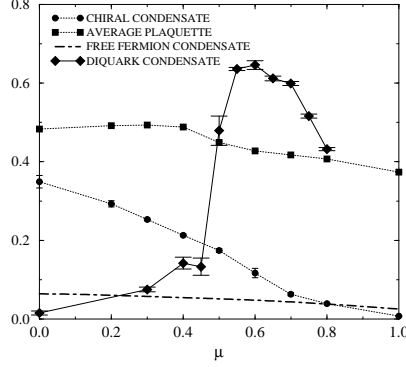


Figure 6.

where the subscripts distinguish between even and odd lattice sites, then due to the pseudoreality of the **2** representation the action may be recast as

$$S_{kin} = \frac{1}{2} \sum_{\text{even}, \mu} \eta_{\mu}(x) [\bar{X}_e(x) U_{\mu}(x) X_o(x + \hat{\mu}) - \bar{X}_e(x) U_{\mu}^{\dagger}(x - \hat{\mu}) X_o(x - \hat{\mu})]. \quad (11)$$

Since the  $X$  and  $\bar{X}$  fields have two components, the global symmetry is enlarged to  $U(2)$ , and can be viewed as relating mesonic  $q\bar{q}$  states to baryonic  $qq$  states. In particular, a condensate  $\langle \bar{\chi}\chi \rangle$ , which in the chiral limit at zero density would be expected to break spontaneously the original  $U(1)_A$  symmetry, can be  $U(2)$ -rotated into a gauge invariant diquark condensate  $\langle qq_2 \rangle \equiv \langle \chi^{tr} \tau_2 \chi \rangle$ , which also breaks  $U(1)_V$ . In either case the overall symmetry breaking is  $U(2) \rightarrow U(1)$ , which is different from the pattern  $SU(2N_f) \rightarrow Sp(2N_f)$  predicted for continuum  $SU(2)$  gauge theory.<sup>13</sup> A full analysis<sup>14</sup> shows that in the limit  $m \rightarrow 0$ ,  $\mu \rightarrow 0$  the symmetry breaking is accompanied by three Goldstone modes, with quantum numbers  $0^-$ ,  $0^+$ ,  $0^+$ . Depending on the direction on the  $U(2)$  manifold chosen by the condensate, the Goldstones can be regarded as either mesons or baryons, and for this reason the presence of baryonic pions in the Monte Carlo simulation is not harmful.<sup>c</sup>

Once  $m$  and  $\mu$  differ from zero, the high degree of symmetry is no longer present. Let us first present some simulation results obtained using a hybrid Monte Carlo (HMC) algorithm, which corresponds to  $N_f = 8$  physical flavors. In Fig. 5 we show results for  $\langle qq_2(j) \rangle$  obtained using eq. (7) for various  $\mu$  on a  $6^4$  lattice with quark mass  $m = 0.05$ , and gauge coupling  $\beta = 1.5$ .<sup>15</sup> There is a

<sup>c</sup> For the model described by the measure  $\det MM^{\dagger}$  the actual pattern is  $U(4) \rightarrow O(4)$ .

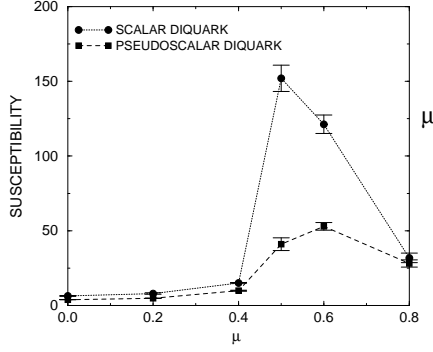


Figure 7.

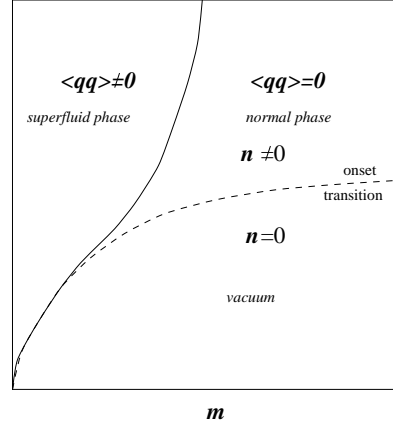


Figure 8.

clear distinction between the curves for  $\mu \leq 0.4$ , which appear to extrapolate to zero as  $j \rightarrow 0$ , and those for  $\mu \geq 0.5$ , which plausibly yield  $\langle qq \rangle \neq 0$  in this limit. Results for  $\langle qq_2 \rangle$  obtained using a simple polynomial extrapolation to the zero source limit are plotted as a function of  $\mu$  in Fig. 6, along with the chiral condensate  $\langle \bar{\chi}\chi \rangle$  and the average plaquette. The chiral condensate appears to decrease smoothly as soon as  $\mu > 0$ , until by  $\mu = 0.8$  it actually falls below its free-field value. The immediate onset of the fall with  $\mu$  is difficult to understand unless there are thermal effects associated with the rather small lattice volume. The diquark condensate, by contrast, stays small (probably even zero within systematic errors) until a value  $\mu_c \simeq 0.4$ , whereupon it rises sharply to a large non-zero value, and then almost immediately starts to fall again. Evidence for a phase transition at  $\mu = \mu_c$  also comes from the average plaquette, which shows a discernible kink between the low density phase, where it is approximately constant, and the high density phase where it decreases with  $\mu$ , presumably as a consequence of Pauli blocking.

It is tempting to associate the decrease in  $\langle qq_2 \rangle$  with  $\mu$  in the dense phase with asymptotic freedom, since the scale defined by the Fermi energy rises monotonically with  $\mu$ , and further measurements reveal that  $n$  also rises significantly over this range of  $\mu$ . It will require much more extensive simulation on a range of lattice volumes and spacings, however, before this effect can be distinguished from lattice artifacts due to saturation.

As mentioned above, the diquark condensate spontaneously breaks the original  $U(1)_V$  symmetry of (9), which remains a symmetry of the Lagrangian even once  $m, \mu \neq 0$ . Therefore we expect the appearance of the condensate



in the dense phase to be accompanied by a scalar Goldstone mode. In Fig. 7 we plot the contributions to the diquark susceptibilities,

$$\chi_{qq} = \sum_x \langle qq(0) \bar{q} \bar{q}(x) \rangle, \quad (12)$$

from connected quark line diagrams in both scalar and pseudoscalar channels. Both increase sharply for  $\mu > \mu_c$ ; the scalar signal, which should project onto an exact Goldstone mode, exceeds the pseudoscalar, which projects onto a pseudo-Goldstone mode. The two channels should become degenerate in the chiral limit.<sup>14</sup> Once again, it is likely that to establish the level ordering in the dense phase unambiguously will require the contribution of disconnected diagrams to be taken into account.<sup>16</sup>

Fig. 8 is a suggested phase diagram for SU(2) lattice gauge theory with fundamental quarks in the  $(m, \mu)$  plane. In principle there are three phases: the *vacuum*, for which both baryon number density  $n$  and diquark condensate  $\langle qq \rangle$  vanish; a *normal* phase for which  $n > 0$  but  $\langle qq \rangle = 0$ ; and a *superfluid* phase<sup>d</sup> (recall  $qq_2$  is gauge invariant) for which both  $n > 0$  and the order parameter  $\langle qq \rangle \neq 0$ . Note that the chiral condensate  $\langle \bar{\chi} \chi \rangle$ , which is usually considered as an order parameter, is everywhere non-zero in this plane, although from Fig. 6 we expect it to decrease from bottom right to top left. The dashed line separating the vacuum from the normal phase is  $\mu_o(m) \propto \sqrt{m}$ , following the arguments about the lightest baryon given above. The solid line  $\mu_c(m)$  which separates the superfluid from the normal phase may well coincide with  $\mu_o$  for some or all of its extent. It will be a goal of future simulations<sup>16</sup> to establish in the first instance whether the normal phase exists, and if so whether it is confined to the large  $m, \mu$  region where  $\langle \bar{\chi} \chi \rangle$  is favoured kinematically, but  $\langle qq \rangle$  suppressed by asymptotic freedom, or extends in a narrow tongue all the way to the zero density chiral limit at the origin. Another issue to explore will be the persistence of the superfluid phase for  $T > 0$ . Finally, even though the model has the wrong physics for  $\mu < \mu_c$ , studies of gluonic dynamics in the dense phase may still be of relevance for QCD;<sup>17</sup> in particular we might expect competition between the enhanced screening expected from a non-zero density of color sources in the ground state, and the kinematic anti-screening due to Pauli blocking.

### 3.2 Adjoint Quarks

The consequences of formulating the theory with adjoint rather than fundamental quarks are profound. Even at zero density, the adjoint model is distinct

<sup>d</sup>Unlike He<sup>3</sup>, superfluidity arises here via BCS pairing in the  $s$ -wave channel.

because of the possibility of gauge invariant spin- $\frac{1}{2}$  bound states, either  $qg$  or  $qqq$ , in the spectrum. We wish to advocate  $SU(2)$  lattice gauge theory with adjoint quarks as a “Toy QCD” for the purposes of non-zero density studies.

First let us discuss the symmetries of the kinetic term for a single staggered flavor. The manipulations leading to eq. (11) go through as before, but now the  $U_\mu$  can be chosen real, and the  $X, \bar{X}$  fields defined:

$$\bar{X}_e = (\bar{\chi}_e, \chi_e^{tr}), \quad X_o^{tr} = (\chi_o^{tr}, \bar{\chi}_o). \quad (13)$$

Formation of a chiral condensate  $\langle \bar{\chi}\chi \rangle$  causes a  $U(2) \rightarrow Sp(2)$  symmetry breaking, resulting in one broken generator whose Goldstone can be identified with the familiar pion. Once again, this is the opposite of the continuum result.<sup>13</sup> Since there are no light diquark states in this case we expect no early onset, ie.  $\lim_{m \rightarrow 0} \mu_o \neq 0$ .

Now consider possible diquark condensates which might form at high density. In the absence of a detailed dynamical argument, we may enunciate three plausible conditions for the  $qq$  operator in the “maximally attractive channel”:

- $qq$  is gauge invariant
- $qq$  is a spacetime scalar
- $qq$  is as local as possible in the lattice  $\chi$  fields

The  $qq_2$  operator discussed in the previous subsection satisfies each of these conditions. For adjoint quarks, however, the Exclusion Principle dictates that one of the conditions must be violated, since the equivalent  $qq_3 \equiv \chi^{tr}(x)\chi(x)$  vanishes identically. A possible non-local chirally symmetric condensate could form from an operator

$$qq'_3 = \sum_{\pm\mu} \eta_\mu(x) (-1)^{x_\mu} [\chi^{tr}(x) U_\mu(x) \chi(x + \hat{\mu}) - \bar{\chi}(x) U_\mu(x) \bar{\chi}^{tr}(x + \hat{\mu})]. \quad (14)$$

This breaks  $U(2) \rightarrow U(1) \otimes U(1)_A$ , resulting in two  $0^+$  Goldstones, one of which persists even once  $m, \mu \neq 0$ . The original  $U(1)_V$  of (9) is broken. A possible phase diagram resulting from  $\langle qq'_3 \rangle$  condensation is shown in Fig. 9. One interesting issue is that although  $qq'_3$  is a scalar under a lattice version of parity,<sup>14</sup> once a transformation is made to fermion fields with continuum degrees of freedom, it is a spacetime pseudoscalar.

A more compelling possibility is condensation of a gauge non-invariant operator

$$qq''_3 = \chi^{tr}(x) \alpha_a t_a \chi(x), \quad (15)$$

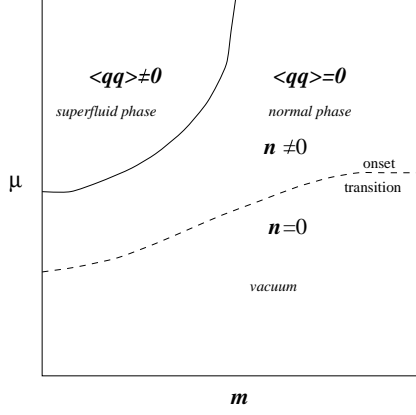


Figure 9.

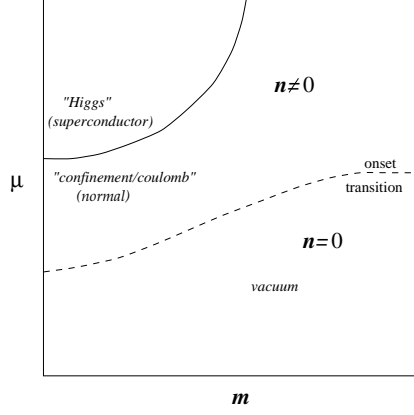


Figure 10.

where the  $t_a$  are generators of the  $\mathbf{3}$ , and are antisymmetric in the representation in which the  $U_\mu$  are real. Because  $qq_{\mathbf{3}}''$  is not gauge singlet, this results in a breaking of the  $SU(2)$  color group to  $U(1)$  by the Higgs mechanism; in other words, this is a superconducting solution. A possible phase diagram is shown in Fig. 10. Note that since  $qq_{\mathbf{3}}''$  acts as an adjoint Higgs field, there is still a separation between normal and superconducting phases,<sup>18</sup> the latter characterised by a massless photon. By contrast, for QCD in the color-flavor locked state, continuity between high and low density phases has been postulated.<sup>19</sup>

We are currently beginning to study the adjoint model using both HMC and multi-bosonic methods.<sup>20</sup> There are two aspects of the simulation to give concern. Firstly, the HMC method permits a minimum  $N_f = 4$ , which destroys asymptotic freedom for adjoint quarks, although that may have little direct bearing on the issue of diquark condensation. Secondly, both algorithms use a power of  $\det M^{tr} M$  which is guaranteed positive, whereas the true measure  $\det M$  is real but not positive definite. In effect, therefore, the simulations incorporate an extra flavor, which allows the possibility of a flavor non-singlet superfluid scalar condensate  $qq_{\mathbf{3}}''' = \chi_i^{tr} \varepsilon_{ij} \chi_j$ . This condensate breaks  $U(4) \rightarrow Sp(4)$  yielding six Goldstones, some of which must be diquark states. Therefore we expect an early onset, and a phase diagram resembling Fig. 8. These considerations point to the intriguing possibility of a link between the sign problem generic to simulations at  $\mu \neq 0$  and a superconducting ground state.

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